HW 4 (Posted on Apr. 9, 2017) **Name:**

HW 4.1 (Sampling distribution)

$$X = \text{ time (in seconds) to react to brake lights during in-traffic driving.}$$

We assume

$$X \sim N(\mu = 2, \sigma^2 = .36).$$

(a) Suppose that we take a random sample of $n = 5$ drivers with times $X_1, \ldots, X_5$.

- What is the distribution of the sample mean $\overline{X}$?
  $\overline{X} \sim N\left(\mu = 2, \sigma^2 = \left(\frac{0.6}{\sqrt{5}}\right)^2\right)$, i.e., $\overline{X} \sim N\left(\mu = 2, \sigma^2 = (0.2683)^2\right)$.
- Is $\overline{X}$ an unbiased estimator of μ?
  Since $E(\overline{X}) = \mu$, $\overline{X}$ is an unbiased estimator of μ.
- Find $P(\overline{X} > 2.1)$.
  $P(\overline{X} > 2.1) = \texttt{normalcdf}(2.1, 10^{99}, 2, 0.2683) = 0.3547$.

(b) Suppose that we take a random sample of $n = 25$ drivers with times $X_1, \ldots, X_{25}$.

- What is the distribution of the sample mean $\overline{X}$?
  $\overline{X} \sim N\left(\mu = 2, \sigma^2 = \left(\frac{0.6}{\sqrt{25}}\right)^2\right)$, i.e., $\overline{X} \sim N\left(\mu = 2, \sigma^2 = 0.12\right)$.
- Is $\overline{X}$ an unbiased estimator of μ?
  Since $E(\overline{X}) = \mu$, $\overline{X}$ is an unbiased estimator of μ.
- Find $P(\overline{X} > 2.1)$.
  $P(\overline{X} > 2.1) = \texttt{normalcdf}(2.1, 10^{99}, 2, 0.12) = 0.2023$.

Bonus! Comparing between (a) and (b), what are your observations?
The larger sample size, the more precious of the population mean.

HW 4.2 (Central limit theorem) The time to death for rate injected with a toxic substance, denoted by X (measured in days), follows an exponential distribution with $\lambda = 1/9$. That is

$$X \sim \text{Exp}(\lambda = 1/9).$$

Suppose that we take a random sample of $n = 36$ drivers with times $X_1, \ldots, X_{36}$.

(a) What is the approximated distribution of $\overline{X}$? (Hint: $\overline{X} \sim AN(\mu, \sigma^2)$. What are mean μ and variance σ^2 ?)

- Note that $E(X) = 1/\lambda = 9$ and $Var(X) = 1/\lambda^2 = 9^2$, according to central limit theorem, we have

$$\overline{X} \sim AN\left(\mu = 9, \sigma = \left(\frac{9}{\sqrt{36}}\right)^2\right),$$

i.e., $\overline{X} \sim AN\left(\mu = 9, \sigma = (1.5)^2\right)$.

(b) Find (or approximate) $P(\overline{X} > 3)$.

- $P(\overline{X} > 3) \approx \texttt{normalcdf}(3, 10^{99}, 9, 1.5) \approx 1$.

HW 4.3 (Quantiles) "Quartiles" are like quarter-based quantiles. We say:

- x is the first quartile (also called the lower quartile) of X's distribution, if $P(X < x) = 0.25$.
- x is the second quartile (also called the median) of X's distribution, if $P(X < x) = 2 \times 0.25 = 0.5$.
- x is the third quartile (also called the upper quartile) of X's distribution, if $P(X < x) = 0.75$.

Find the three quartiles of X if X follows:

(a) $N(0,1)$

- First quantile = `invNorm`$(0.25, 0, 1) = -0.6749$.
- Second quantile = `invNorm`$(0.5, 0, 1) = 0$.
- Thrid quantile = `invNorm`$(0.75, 0, 1) = 0.6749$.

(b) $N(1,1)$

- First quantile = `invNorm`$(0.25, 1, 1) = 0.3255$.
- Second quantile = `invNorm`$(0.5, 1, 1) = 1$.
- Third quantile = `invNorm`$(0.75, 1, 1) = 1.6749$.

(c) $N(2,1)$

- First quantile = `invNorm`$(0.25, 2, 1) = 1.3255$.
- Second quantile = `invNorm`$(0.5, 2, 1) = 2$.
- Third quantile = `invNorm`$(0.75, 2, 1) = 2.6749$.

Bonus! Can you find a pattern? (all these distributions are normal with a same variance, means are increasing by 1, does this increasing pattern affect the quartiles?)

• Since the above distributions are shifted according to shifting of mean, so do corresponding quantile values.

HW 4.4 (Confidence intervals of population mean μ when σ is known.) Past experience has indecated that the breaking strength of yarn used in manufacturing drapery material is normally distributed and that $\sigma = 2$ psi. A random sample of nine specimens is tested, and the average breaking strength is found to be 92.514 psi.

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98.73 91.27 93.53 88.61 92.06 89.23 94.19 92.67 92.34
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(a) Construct 90%, 95%, and 99% two-sided confidence intervals on the true mean breaking strength μ. Compare the length of these intervals, tell me your conclusion.

- 90% confidence interval: $(91.42, 93.61)$.
- 95% confidence interval: $(91.21, 93.82)$.
- 99% confidence interval: $(90.80, 94.23)$.

Given the same information that data provided, in order to be more confident, we need a larger interval to cover unobserved μ.

(b) Give an interpretation of the 95% confidence interval on the true mean breaking strength μ.
• We are 95% confident that the true mean breaking strength μ is between 91.21 psi and 93.82 psi.

(c) Construct a 95% lower one-sided confidence interval on the true mean breaking strength μ and give an interpretation.
• 95% lower-confidence bound: $(91.42, \infty)$. We are 95% confident that the true mean breaking strength is larger than 91.42.

(d) Construct a 95% upper one-sided confidence interval on the true mean breaking strength μ and give an interpretation.
• 95% upper-confidence bound: $(-\infty, 93.61)$. We are 95% confident that the true mean breaking strength is smaller than 93.61.

(e) If the scientists want the confidence interval to be no wider than 0.5 psi, how many observations should they take? (Note that, the width of the confidence interval is two times the margin error E, so $E = 0.5/2$).

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.96(2)}{0.25}\right)^2 \approx 246.$$

The scientists need at least 246 observations.

HW 4.5. **(Confidence intervals of population mean μ when σ is unknown.)** Ishikawa et al. (Journal of Bioscience and Bioengineering, 2012) studied the adhesion of various biofilms to solid surfaces for possible use in environmental technologies. Adhesion assay is conducted by measuring absorbance at A590. Suppose that for the bacterial strain Acinetobacter, five measurements gave readings of 2.69, 5.76, 2.67, 1.62 and 4.12 dyne-cm^2. Assume that the population distribution is normal.

(a) Find a 95% confidence interval for the mean adhesion and give an interpretation.

- Two-sided 95% confidence interval: $(1.38, 5.35)$. We are 95% confident that the population mean adhesion is between 1.38 and 5.35.

(b) Find a 95% confidence interval for the mean adhesion by using sample mean $\overline{x} = 3.372$ and sample standard deviation $s = 1.604$.

- Since $t_{4,0.025} = 2.78$, then we have a 95% confidence interval:

$$\left(\overline{X} - t_{(n-1),\alpha/2}\frac{S}{\sqrt{n}}, \overline{X} + t_{(n-1),\alpha/2}\frac{S}{\sqrt{n}}\right) = \left(3.37 - 2.78\frac{1.60}{\sqrt{5}}, 3.37 + 2.78\frac{1.60}{\sqrt{5}}\right) = (1.38, 5.35).$$

(c) If the scientists want the confidence interval to be no wider than 0.5 dyne-cm^2, how many observations should they take? (Note that, the width of the confidence interval is two times the margin error E, so $E = 0.5/2$).

- Not solvable.

(d) Find the 95% lower and upper confidence bounds for the mean adhesion and give interpretations.

- 95% lower-confidence bound: $(1.84, \infty)$. We are 95% confident that the population mean adhesion is larger than 1.84.
- 95% upper-confidence bound: $(-\infty, 4.9)$. We are 95% confident that the population mean adhesion is less than 4.90.

HW 4.6. (Confidence intervals of population proportion.) A random sample of 50 suspension helmets used by motorcycle riders and automobile race-car drivers was subjected to an impact test, and some damage was observed on 18 of these helmets.

(a) Find a 95% two-sided confidence interval on the true proportion of helmets, denoted by p, that would show damage from this test and give an interpretation.
• 95% two-sided confidence interval: $(0.2270, 0.4930)$. We are 95% confident that the population proportion of helmets that would show damage from the test is between 0.2270 and 0.4930.

(b) Find 95% lower and upper one-sided confidence bounds on the true proportion of helmets, denoted by p, that would show damage from this test and give interpretation.
• 95% lower-confidence bound: $(0.2453, 1)$. We are 95% confident that the population proportion of helmets that would show damage from the test is large than 0.2453.
• 95% upper-confidence bound: $(0, 0.4747)$. We are 95% confident that the population proportion of helmets that would show damage from the test is smaller than 0.4747.

(c) Using the point estimate of p from the 50 helmets, how many helmets must be tested to be 95% confident that the marginal error E is less than 0.02?
• Guessing $p_0 = \hat{p} = 0.36$, then

$$n = \left(\frac{z_{\alpha/2}}{E}\right) p_0(1 - p_0) \approx 2213.$$

(d) How large must the sample be if we wish to be at least 95% confident that the marginal error E is less than 0.02 regardless of the true value of p?
• Given the most conservative situation, $p_0 = 1/2$, then

$$n = \left(\frac{z_{\alpha/2}}{E}\right) \frac{1}{2} \left(1 - \frac{1}{2}\right) \approx 2401.$$

HW 4.7 (Confidence interval of population variance and standard deviation.) An article in Cancer Research [Analyses of Litter Matched Time-to-Response Data, with Modifications for Recovery of Interlitter Information (1977, Vol. 37, pp. 38633868)] tested the tumorigenesis of a drug. Rats were randomly selected from litters and given the drug. The times of tumor appearance were recorded as follows:
101, 104, 104, 77, 89, 88, 104, 96, 82, 70, 89, 91, 39, 103, 93, 85, 104, 104, 81, 67, 104, 104, 104, 87, 104, 89, 78, 104, 86, 76, 103, 102, 80, 45, 94, 104, 104, 76, 80, 72, 73

- Please calculate a 95% confidence interval on the population variance of time and give an interpretation by following steps:
  (1) Determine significant level α and sample size n. **Ans.** $\alpha = 0.05$ and $n = 41$.
  (2) Find the sample mean $\overline{x}$ and sample standard deviation s. **Ans.** $\overline{x} = 88.78$ and $s = 15.99$.
  (3) Find the upper $(\alpha/2)$th, $(1-\alpha/2)$th, αth, and $(1-\alpha)$ quantile values, $\chi^2_{n-1,\alpha/2}$, $\chi^2_{n-1,1-\alpha/2}$, $\chi^2_{n-1,\alpha}$, and $\chi^2_{n-1,1-\alpha}$, respectively. (Hint: χ^2 distribution table on page 9.) **Ans.** $\chi^2_{n-1,\alpha/2} = \chi_{40,0.025} = 59.342$, $\chi^2_{n-1,1-\alpha/2} = \chi_{40,0.975} = 24.433$, $\chi^2_{n-1,\alpha} = \chi_{40,0.05} = 55.758$, and $\chi^2_{n-1,1-\alpha} = \chi_{40,0.95} = 26.502$.
  (4) Calculate the 95% confidence interval by
  $$\left(\frac{(n-1)S^2}{\chi^2_{n-1,\alpha/2}}, \frac{(n-1)S^2}{\chi^2_{n-1,1-\alpha/2}}\right) = (172.41, 418.74).$$
  (5) Please give an interpretation of the 95% confidence interval.
  We are 95% confident that the population variance is between 172.41 and 418.74.

- According to the result above, calculate a 95% confidence interval on the population standard deviation of time. Hint: The standard deviation is the square-root of variance. Then we should have:
  $$\left(\sqrt{\frac{(n-1)S^2}{\chi^2_{n-1,\alpha/2}}}, \sqrt{\frac{(n-1)S^2}{\chi^2_{n-1,1-\alpha/2}}}\right) = (13.13, 20.46)$$

- Calculate a 95% lower one-sided confidence interval on the population standard deviation of time. Hint:
  $$\left(\sqrt{\frac{(n-1)S^2}{\chi^2_{n-1,\alpha}}},\ \infty\right) = (13.54, \infty).$$

- Calculate a 95% upper one-sided confidence interval on the population standard deviation of time. Hint:
  $$\left(0, \sqrt{\frac{(n-1)S^2}{\chi^2_{n-1,1-\alpha}}}\right) = (0, 19.65).$$

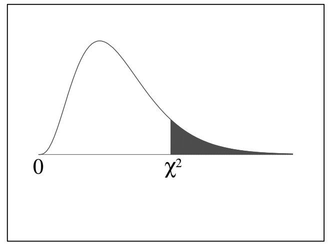
t Table upper-tail probability:

df	.25	.10	.05	.025	.01	.005
1	1.0000	3.0777	6.314	12.706	31.821	63.657
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.925
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0321
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1314	2.6025	2.9467
16	0.6901	1.3368	1.7459	2.1199	2.5835	2.9208
17	0.6892	1.3334	1.7396	2.1098	2.5669	2.8982
18	0.6884	1.3304	1.7341	2.1009	2.5524	2.8784
19	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
20	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
21	0.6864	1.3232	1.7207	2.0796	2.5176	2.8314
22	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
23	0.6853	1.3195	1.7139	2.0687	2.4999	2.8073
24	0.6848	1.3178	1.7109	2.0639	2.4922	2.7969
25	0.6844	1.3163	1.7081	2.0595	2.4851	2.7874
26	0.6840	1.3150	1.7056	2.0555	2.4786	2.7787
27	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
28	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
29	0.6830	1.3114	1.6991	2.0452	2.4620	2.7564
30	0.6828	1.3104	1.6973	2.0423	2.4573	2.7500
31	0.6825	1.3095	1.6955	2.0395	2.4528	2.7440
32	0.6822	1.3086	1.6939	2.0369	2.4487	2.7385
33	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
34	0.6818	1.3070	1.6909	2.0322	2.4411	2.7284
35	0.6816	1.3062	1.6896	2.0301	2.4377	2.7238
36	0.6814	1.3055	1.6883	2.0281	2.4345	2.7195
37	0.6812	1.3049	1.6871	2.0262	2.4314	2.7154
38	0.6810	1.3042	1.6860	2.0244	2.4286	2.7116
39	0.6808	1.3036	1.6849	2.0227	2.4258	2.7079
40	0.6807	1.3031	1.6839	2.0211	2.4233	2.7045
41	0.6805	1.3025	1.6829	2.0195	2.4208	2.7012
42	0.6804	1.3020	1.6820	2.0181	2.4185	2.6981
43	0.6802	1.3016	1.6811	2.0167	2.4163	2.6951
44	0.6801	1.3011	1.6802	2.0154	2.4141	2.6923
45	0.6800	1.3006	1.6794	2.0141	2.4121	2.6896
46	0.6799	1.3002	1.6787	2.0129	2.4102	2.6870
47	0.6797	1.2998	1.6779	2.0117	2.4083	2.6846
48	0.6796	1.2994	1.6772	2.0106	2.4066	2.6822
49	0.6795	1.2991	1.6766	2.0096	2.4049	2.6800
50	0.6794	1.2987	1.6759	2.0086	2.4033	2.6778

t Table upper-tail probability:

df	.25	.10	.05	.025	.01	.005
51	0.6793	1.2984	1.6753	2.0076	2.4017	2.6757
52	0.6792	1.2980	1.6747	2.0066	2.4002	2.6737
53	0.6791	1.2977	1.6741	2.0057	2.3988	2.6718
54	0.6791	1.2974	1.6736	2.0049	2.3974	2.6700
55	0.6790	1.2971	1.6730	2.0040	2.3961	2.6682
56	0.6789	1.2969	1.6725	2.0032	2.3948	2.6665
57	0.6788	1.2966	1.6720	2.0025	2.3936	2.6649
58	0.6787	1.2963	1.6716	2.0017	2.3924	2.6633
59	0.6787	1.2961	1.6711	2.0010	2.3912	2.6618
60	0.6786	1.2958	1.6706	2.0003	2.3901	2.6603
61	0.6785	1.2956	1.6702	1.9996	2.3890	2.6589
62	0.6785	1.2954	1.6698	1.9990	2.3880	2.6575
63	0.6784	1.2951	1.6694	1.9983	2.3870	2.6561
64	0.6783	1.2949	1.6690	1.9977	2.3860	2.6549
65	0.6783	1.2947	1.6686	1.9971	2.3851	2.6536
66	0.6782	1.2945	1.6683	1.9966	2.3842	2.6524
67	0.6782	1.2943	1.6679	1.9960	2.3833	2.6512
68	0.6781	1.2941	1.6676	1.9955	2.3824	2.6501
69	0.6781	1.2939	1.6672	1.9949	2.3816	2.6490
70	0.6780	1.2938	1.6669	1.9944	2.3808	2.6479
71	0.6780	1.2936	1.6666	1.9939	2.3800	2.6469
72	0.6779	1.2934	1.6663	1.9935	2.3793	2.6459
73	0.6779	1.2933	1.6660	1.9930	2.3785	2.6449
74	0.6778	1.2931	1.6657	1.9925	2.3778	2.6439
75	0.6778	1.2929	1.6654	1.9921	2.3771	2.6430
76	0.6777	1.2928	1.6652	1.9917	2.3764	2.6421
77	0.6777	1.2926	1.6649	1.9913	2.3758	2.6412
78	0.6776	1.2925	1.6646	1.9908	2.3751	2.6403
79	0.6776	1.2924	1.6644	1.9905	2.3745	2.6395
80	0.6776	1.2922	1.6641	1.9901	2.3739	2.6387
81	0.6775	1.2921	1.6639	1.9897	2.3733	2.6379
82	0.6775	1.2920	1.6636	1.9893	2.3727	2.6371
83	0.6775	1.2918	1.6634	1.9890	2.3721	2.6364
84	0.6774	1.2917	1.6632	1.9886	2.3716	2.6356
85	0.6774	1.2916	1.6630	1.9883	2.3710	2.6349
86	0.6774	1.2915	1.6628	1.9879	2.3705	2.6342
87	0.6773	1.2914	1.6626	1.9876	2.3700	2.6335
88	0.6773	1.2912	1.6624	1.9873	2.3695	2.6329
89	0.6773	1.2911	1.6622	1.9870	2.3690	2.6322
90	0.6772	1.2910	1.6620	1.9867	2.3685	2.6316
91	0.6772	1.2909	1.6618	1.9864	2.3680	2.6309
92	0.6772	1.2908	1.6616	1.9861	2.3676	2.6303
93	0.6771	1.2907	1.6614	1.9858	2.3671	2.6297
94	0.6771	1.2906	1.6612	1.9855	2.3667	2.6291
95	0.6771	1.2905	1.6611	1.9853	2.3662	2.6286
96	0.6771	1.2904	1.6609	1.9850	2.3658	2.6280
97	0.6770	1.2903	1.6607	1.9847	2.3654	2.6275
98	0.6770	1.2902	1.6606	1.9845	2.3650	2.6269
99	0.6770	1.2902	1.6604	1.9842	2.3646	2.6264
100	0.6770	1.2901	1.6602	1.9840	2.3642	2.6259
110	0.6767	1.2893	1.6588	1.9818	2.3607	2.6213
120	0.6765	1.2886	1.6577	1.9799	2.3578	2.6174
∞	0.6745	1.2816	1.6449	1.9600	2.3264	2.5758

Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi^2_\alpha$.

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169